Exam 4 Review

1. $(4x^{-2}y^{-7})(5xy^{-3})$
$= 20x^{-1}y^{-10}$
$= \boxed{\dfrac{20}{xy^{10}}}$

2. $\dfrac{36a^{-2}b^{5}}{4a^{-6}b^{9}} = 9a^{-2-(-6)}b^{5-9} = 9a^{4}b^{-4} = \boxed{\dfrac{9a^4}{b^4}}$

3. $\left(\dfrac{3x^{2}y^{-5}}{x^{-6}y^{2}}\right)^{-2} = \left(\dfrac{3x^{8}}{y^{7}}\right)^{-2} = \left(\dfrac{y^{7}}{3x^{8}}\right)^{2} = \boxed{\dfrac{y^{14}}{9x^{16}}}$

4. $\sqrt{81x^2} = |9x| = \boxed{9|x|}$

9x could be negative, so you need absolute value

5. $\sqrt{64y^4} = \boxed{8y^2}$

$8y^2$ is always positive, so you don't need absolute value.

6. $f(x) = \sqrt[3]{x+5}$
Domain: $\mathbb{R}$

You can take an odd root of anything

7. $f(x) = \sqrt[4]{2x-6}$
$2x-6 \geq 0$
$2x \geq 6$
$x \geq 3$ $\quad \boxed{[3,\infty)}$

Can't take an even root of a negative, so radicand ≥ 0.

8. $f(x) = \sqrt[3]{x-5}$
$f(-22) = \sqrt[3]{-22-5} = \sqrt[3]{-27} = \boxed{-3}$

9. $f(x) = \sqrt[4]{x-5}$
$f(-620) = \sqrt[4]{-620-5} = \sqrt[4]{-625}$ $\quad$ $\boxed{\text{Does not exist}}$

10. $(x^5y^6)^{1/7} = \boxed{\sqrt[7]{x^5y^6}}$

11. $(4x^6)^{3/2} = (\sqrt{4x^6})^3 = (2x^3)^3 = \boxed{8x^9}$

12. $(\sqrt[5]{6xy})^3 = \boxed{(6xy)^{3/5}}$

13. $\sqrt[4]{\dfrac{ab^3}{7}} = \boxed{\left(\dfrac{ab^3}{7}\right)^{1/4}}$

14. $4^{4/5} \cdot 4^{1/3} = 4^{4/5 + 1/3} = 4^{12/15 + 5/15} = \boxed{4^{17/15}}$

15. $(x^{-2/3}y^{3/8})^{1/2} = \left(\dfrac{y^{3/8}}{x^{2/3}}\right)^{1/2} = \boxed{\dfrac{y^{3/16}}{x^{1/3}}}$

16. $\sqrt{6q}\,\sqrt{5p} = \boxed{\sqrt{30pq}}$

17. $\sqrt{12x^5y^4} = \sqrt{2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y}$

$= 2 \cdot x \cdot x \cdot y \cdot y \sqrt{3x}$

$= \boxed{2x^2y^2\sqrt{3x}}$

18. $\sqrt[4]{x^{12}y^9z^{27}} =$

$\sqrt[4]{x^{12}y^8z^{24}} \cdot \sqrt[4]{yz^3}$

$= \boxed{x^3y^2z^6\sqrt[4]{yz^3}}$

19. $f(x) = \sqrt{36(x-7)^2}$

$f(x) = \sqrt{36} \cdot \sqrt{(x-7)^2}$

$\boxed{f(x) = 6|x-7|}$

since x-7 could be negative,
you need absolute value.

20. $\dfrac{\sqrt{48xy^5}}{\sqrt{6x}} = \sqrt{\dfrac{48xy^5}{6x}} = \sqrt{8y^5} = \sqrt{4y^4} \cdot \sqrt{2y} = \boxed{2y^2\sqrt{2y}}$

21. $\sqrt{\dfrac{75}{x}} = \sqrt{\dfrac{75 \cdot x}{x \cdot x}} = \dfrac{\sqrt{75x}}{x} = \dfrac{\sqrt{25} \cdot \sqrt{3x}}{x} = \boxed{\dfrac{5\sqrt{3x}}{x}}$

22. $\sqrt[3]{\dfrac{7}{4y}} = \sqrt[3]{\dfrac{7}{2\cdot 2\cdot y}} = \sqrt[3]{\dfrac{7}{2\cdot 2\cdot y}\cdot\dfrac{2\cdot y\cdot y}{2\cdot y\cdot y}} = \sqrt[3]{\dfrac{14y^2}{2^3y^3}} = \boxed{\dfrac{\sqrt[3]{14y^2}}{2y}}$

23. $8\sqrt{3} - 10\sqrt{27} =$

27
9 3
3 3

$8\sqrt{3} - 10\cdot 3\sqrt{3} =$

$8\sqrt{3} - 30\sqrt{3} =$

$\boxed{-22\sqrt{3}}$

24. $5\sqrt[4]{x^6} - 6x\sqrt[4]{x^2} =$

$5\sqrt[4]{x^4}\cdot\sqrt[4]{x^2} - 6x\sqrt[4]{x^2} =$

$5x\sqrt[4]{x^2} - 6x\sqrt[4]{x^2} =$

$\boxed{-x\sqrt[4]{x^2}}$

25. $\sqrt{3}\left(\sqrt{12} - \sqrt{3}\right) = \sqrt{36} - \sqrt{9} = 6 - 3 = \boxed{3}$

26. $(\sqrt{7} + 2)(\sqrt{5} - 4) =$

$\boxed{\sqrt{35} - 4\sqrt{7} + 2\sqrt{5} - 8}$

FOIL
Don't forget the "OI"!

27. $\dfrac{5-\sqrt{3}}{5+\sqrt{3}} = \left(\dfrac{5-\sqrt{3}}{5+\sqrt{3}}\right)\left(\dfrac{5-\sqrt{3}}{5-\sqrt{3}}\right)$

$= \dfrac{25 - 5\sqrt{3} - 5\sqrt{3} + 3}{5^2 - (\sqrt{3})^2} = \dfrac{28 - 10\sqrt{3}}{25 - 3}$

$= \dfrac{28 - 10\sqrt{3}}{22} = \dfrac{2(14 - 5\sqrt{3})}{22}$

$= \boxed{\dfrac{14 - 5\sqrt{3}}{11}}$

28. $\dfrac{\sqrt[4]{x}}{\sqrt[3]{x^2}} = \dfrac{x^{1/4}}{x^{2/3}} = \dfrac{x^{3/12}}{x^{8/12}} = x^{-5/12} = \dfrac{1}{x^{5/12}} = \boxed{\dfrac{1}{\sqrt[12]{x^5}}}$

29. $\sqrt{4x} + 3 = 8$

$\sqrt{4x} = 5$

$(\sqrt{4x})^2 = 5^2$

$4x = 25$

$\boxed{x = 25/4}$

Check: $\sqrt{4 \cdot 25/4} + 3 \stackrel{?}{=} 8$

$\sqrt{25} + 3 \stackrel{?}{=} 8$

$5 + 3 = 8$ ✓

30. $y^{1/3} + 3 = -1$

$y^{1/3} = -4$

$(y^{1/3})^3 = (-4)^3$

$\boxed{y = -64}$

Check: $(-64)^{1/3} + 3 = -1$

$-4 + 3 = -1$ ✓

31. $\sqrt{x+2} + \sqrt{3x+4} = 2$

$\sqrt{x+2} = 2 - \sqrt{3x+4}$

$(\sqrt{x+2})^2 = (2 - \sqrt{3x+4})^2$

$x + 2 = -4\sqrt{3x+4} + 3x + 8$

$-3x - 8 \qquad\qquad -3x - 8$

$-2x - 6 = -4\sqrt{3x+4}$

$(-2x-6)^2 = (-4\sqrt{3x+4})^2$

$4x^2 + 24x + 36 = 16(3x+4)$

$4x^2 + 24x + 36 = 48x + 64$

$-48x - 64 \qquad -48x - 64$

$4x^2 - 24x - 28 = 0$

$\frac{\cancel{4}(x^2 - 6x - 7)}{\cancel{4}} = \frac{0}{4}$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$x - 7 = 0$ or $x + 1 = 0$

$\cancel{x = 7}$ or $\boxed{x = -1}$

$(2 - \sqrt{3x+4})(2 - \sqrt{3x+4})$

$= 4 - 2\sqrt{3x+4} - 2\sqrt{3x+4} + 3x + 4$

$= -4\sqrt{3x+4} + 3x + 8$

$(-2x-6)(-2x-6)$

$= 4x^2 + 12x + 12x + 36$

$= 4x^2 + 24x + 36$

Check: $\sqrt{7+2} + \sqrt{3(7)+4} \stackrel{?}{=} 2$

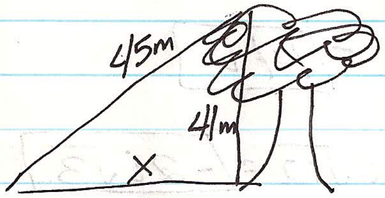
$\sqrt{9} + \sqrt{25} \stackrel{?}{=} 2$

$3 + 5 \neq 2$

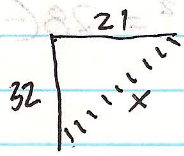
$\sqrt{-1+2} + \sqrt{3(-1)+4} \stackrel{?}{=} 2$

$\sqrt{1} + \sqrt{1} \stackrel{?}{=} 2$

$1 + 1 = 2$ ✓

32. 

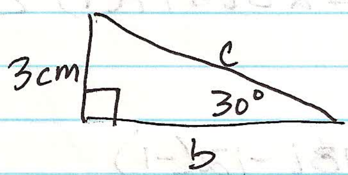
$$x^2 + 41^2 = 45^2$$
$$x^2 = 45^2 - 41^2$$
$$x = \sqrt{45^2 - 41^2} = \boxed{18.5\text{ m}}$$

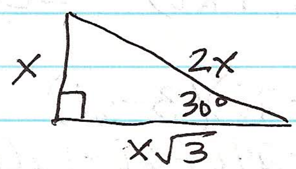
33. 

$$32^2 + 21^2 = x^2$$
$$x = \sqrt{32^2 + 21^2} = 38.3\text{ ft}$$

Distance using separate crosswalks: $32 + 21 = 53$ ft

Distance saved by using diagonal crosswalk: $53 - 38.3 = \boxed{14.7\text{ft}}$

34. 



$b = 3\sqrt{3}$

$c = 2(3)$

$\boxed{b = 5.2\text{ cm}}$
$\boxed{c = 6\text{ cm}}$

35. $(2, -4)$ $(-5, 3)$ distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(-5-2)^2 + (3-(-4))^2}$$
$$d = \sqrt{(-7)^2 + 7^2}$$
$$d = \sqrt{49+49} = \sqrt{98} \approx \boxed{9.899}$$

36. $(2, -5)$ $(-1, -9)$ midpoint

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + (-1)}{2}, \frac{-5 + (-9)}{2}\right)$$
$$\boxed{\left(\tfrac{1}{2}, -7\right)}$$

37. $\sqrt{-25} + \sqrt{-81} = 5i + 9i = \boxed{14i}$

38. $-\sqrt{-12} = -\sqrt{-1} \cdot \sqrt{12} = -i\sqrt{12} = \boxed{-2i\sqrt{3}}$

12
4 3
2 2

39. $\sqrt{-16} \cdot \sqrt{-49} = 4i \cdot 7i = 28i^2 = 28(-1) = \boxed{-28}$

40. $(4+2i) + (3-6i) = \boxed{7-4i}$

41. $(4+2i) - (3-6i) =$
$4 + 2i - 3 + 6i = \boxed{1+8i}$

42. $7i(4+2i) = 28i + 14i^2 = 28i + 14(-1) = \boxed{-14+28i}$

43. $(4+2i)(3-6i)$
$= 12 - 24i + 6i - 12(i^2) = 12 - 18i - 12(-1)$
$= 12 - 18i + 12 = \boxed{24-18i}$

44. $(3-6i)^2 = (3-6i)(3-6i)$
$= 9 - 18i - 18i + 36i^2 = 9 - 36i + 36(-1)$
$= \boxed{-27-36i}$

45. $(3+6i)(3-6i)$ $\qquad (a+bi)(a-bi) = a^2 + b^2$
$= 3^2 + 6^2 = 9 + 36 = \boxed{45}$

46. $\left(\frac{4+2i}{3-6i}\right)\left(\frac{3+6i}{3+6i}\right) = \frac{12 + 24i + 6i + 12i^2}{3^2 + 6^2} = \frac{12 + 30i - 12}{45}$
$= \frac{30i}{45} = \boxed{\frac{2}{3}i}$

47. $i^{32} = (i^2)^{16} = (-1)^{16} = \boxed{1}$ $\qquad$ 48. $i^{21} = (i^2)^{10} \cdot i = (-1)^{10} \cdot i = \boxed{i}$

49. $i^{42} = (i^2)^{21} = (-1)^{21} = \boxed{-1}$ $\qquad$ 50. $i^{39} = (i^2)^{19} \cdot i = (-1)^{19} \cdot i = \boxed{-i}$